Interaction Energy of a Charged Medium and its EM Field in a Curved Spacetime

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ABSTRACT

In special relativity (SR) as well as in general relativity (GR), the energy(-momentum-stress) tensors of a charged continuum and its electromagnetic field: $\boldsymbol{T}_{\text{chg}}$ and $\boldsymbol{T}_{\text{field}}$, can be assumed to add to give the total energy tensor $\boldsymbol{T}$ that obeys the standard dynamical equation

$$T^{\mu\nu}_{\;\;;\nu} = 0. \tag{1}$$

That is, in SR and in GR, one may assume

$$\boldsymbol{T} = \boldsymbol{T}_{\text{chg}} + \boldsymbol{T}_{\text{field}}. \tag{2}$$

In fact, by using (2), the second Maxwell group of GR can be *derived* from (1) and the equation of motion of the charged continuum (instead of using the equivalence principle to go from the equation of SR to that of GR), as we show in this work. The same can be done in the preferred-frame theory of gravitation based on a scalar field ("scalar ether theory" or SET, for which an application of the standard equivalence principle is precluded) [1]. However, for SET, this way of closing the equation system of electrodynamics turns out to be not satisfying, mainly because it leads to charge production/destruction at untenable rates, and also because it has two other doubtful theoretical consequences [2]. Therefore, we must abandon the additivity assumption (2), which means to introduce an "interaction energy tensor" $\boldsymbol{T}_{\text{inter}}$ such that

$$\boldsymbol{T} = \boldsymbol{T}_{\text{chg}} + \boldsymbol{T}_{\text{field}} + \boldsymbol{T}_{\text{inter}}. \tag{3}$$

In order to precise the constraints that are imposed on $\boldsymbol{T}_{\text{inter}}$, we investigate in some detail the independent equations of continuum electrodynamics and their number, beginning with the case of GR. We show that, for GR and for SET as well, the system made of Maxwell's first group and the equation of motion of the charged continuum, plus Eq. (1), is indeed closed under the additivity assumption (2). While introducing $\boldsymbol{T}_{\text{inter}}$ by switching to (3), one necessarily introduces new unknowns, so that the foregoing system is not closed any more

and one needs new equations. We impose on $\boldsymbol{T}_{\text{inter}}$ that, in SR, it should be a Lorentz-invariant tensor. This determines that in the general case it has the form $T^{\mu}_{\text{inter}\ \ \nu} := p\delta^{\mu}_{\nu}$, with p a scalar field, which we show is constant in SR. We show that the additional equation can then consistently be imposed to be th e charge conservation. Finally, we derive the equations that are then got to determine the scalar field p, hence the interaction energy, and we indicate how one may in principle compute that energy in a given EM field and with a given distribution of matter. The interaction energy would then count as "dark matter".

References

[1] Arminjon M., *Continuum Dynamics and the Electromagnetic Field in the Scalar Ether Theory of Gravitation*, Open Physics **14** (2016) 395-409 .

[2] Arminjon M., *Charge Conservation in a Gravitational Field in the Scalar Ether Theory*, Open Physics **15** (2017) 877-890.