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EXACT NONLINEAR EQUATIONS FOR FLUID FILMS AND PROPER ADAPTATIONS OF CONSERVATION THEOREMS FROM CLASSICAL HYDRODYNAMICS

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Abstract. We discuss the exact nonlinear equations for the dynamics of fluid films, modeled as a two dimensional manifold. Our main goal is to illustrate the differences and similarities between the fluid film equations and Euler's equations, their classical three dimensional counterpart. Since the geometry of fluid films is fundamentally different – three dimensional velocity field on a two dimensional support with a time varying Riemannian metric – all classical theorems must be properly modified. We offer adaptations of the following theorems: conservation of mass and energy, pointwise conservation of vorticity and Kelvin's circulation theorem. We present proofs of these theorems by employing the calculus of moving surfaces. It is of great interest to develop a simplified model that captures normal deformations of fluid films by assuming that tangential velocities vanish while preserving the exact nonlinear nature of the full system. This cannot be accomplished simply by neglecting the tangential components, for such an attempt leads to internal contradictions. Instead, we modify the initial formulation and present a modified variational approach that leads to a simplified system of equations capable of capturing a broad range of deeply nonlinear effects.

1. Introduction

Fluid dynamics is one of the most developed subjects in classical physics [1], [2] and still one of the most active today. Fluid films have always occupied an important place in hydrodynamics [3], [4] and have recently been receiving a great deal of renewed attention [5–11]. In this paper, we discuss the exact nonlinear equations for fluid films under the influence of generalized surface tension. Laplace's classical model of surface tension figures is a special case.

Historically, the governing equations of fluid dynamics were formulated by analogy with Newton's laws of motion. The force F is postulated as a function of geometry or kinetics. An attempt to do the same for fluid films would be certainly