



GEOMETRY OF MEMBRANES

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Abstract. This review reports some theoretical results on the geometry of membranes. The governing equations to describe equilibrium configurations of lipid vesicles, lipid membranes with free edges, and chiral lipid membranes are derived from the variation of free energies of these structures. Some analytic solutions to these equations and their corresponding configurations are also shown.

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