JGSP **28** (2012) 105–112

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

CLAIRAUT'S THEOREM IN MINKOWSKI SPACE

ANIS SAAD AND ROBERT J. LOW

Communicated by Gregory L. Naber

Abstract. We consider some aspects of the geometry of surfaces of revolution in three-dimensional Minkowski space. First, we show that Clairaut's theorem, which gives a well-known characterization of geodesics on a surface of revolution in Euclidean space, has an analogous result in three-dimensional Minkowski space. We then illustrate the significant differences between the two cases which arise in spite of their formal similarity.

1. Introduction

The relationship between Euclidean and Minkowskian geometry has many intriguing aspects, one of which is the manner in which formal similarity can co-exist with significant geometric disparity. There has been considerable interest in the comparison of these two geometries, as we see from the lecture notes of López [3]. In particular, aspects of surfaces of revolution in Minkowski space have been considered, *e.g.* in [2]. There is an elegant characterization of godesics on surfaces of revolution due to Clairaut–see, for example, Pressley's differential geometry textbook [7], which is a valuable tool in the study of such surfaces in the Euclidean context [1, 4–6]. Our purpose here is to see how this characterization carries over to Minkowski space, and how it can be used to investigate the difference between the two situations.

2. Euclidean Geometry

We begin by recalling the situation in Euclidean space, the better to see how closely the situation in Minkowski space parallels this one.

Let Σ be a surface of revolution, obtained by rotating the profile curve $x = \rho(u)$, $y = 0$, $z = h(u)$ about the axis of symmetry, where we assume that $\rho > 0$ and

doi: 10.7546/jgsp-28-2012-105-112