Geometry, Integrability and Quantization
September 1–10, 1999, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2000, pp 229-240

INVARIANT REDUCTION OF THE TWO-BODY PROBLEM WITH CENTRAL INTERACTION ON SIMPLY CONNECTED SPACES OF CONSTANT SECTIONAL CURVATURE

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Abstract. The problem of two classical particles with central interaction on simply connected spaces of a constant curvature is considered from the invariant point of view. The Hamiltonian reduction method is used for exluding a movement of the system as a whole.

1. Introduction

Everybody knows that the two body problem with central interaction in Euclidean space of an arbitrary dimension is reduced to the problem of one body in a central potential. On the other hand there exist spaces of a constant sectional curvature, which possess as wide isometry group as the Euclidean space of the same dimension and are homogeneous and isotropic. In this connection the following question arises: What is the most effective way of using their isometry group for the simplifying of the two-body problem on spaces of the constant sectional curvature?

Detail analysis shows that for the last problem there is no an analog of the Galilei transformation and a naturally defined center of mass doesn't move along a geodesic even in the case without interaction. So, for simplifying this problem we can use only the isometry group. It had been shown [1] that the exluding of a movement of the center of mass for n-particle system in Euclidean space can be carried out by the Marsden–Weinstein reduction method. Obviously the result is the same as after the using for the same purpose the Galilei transformation.

The Hamiltonian reduction had been used for the classical two-body problem in the spaces of a constant sectional curvature [2]. This reduction was based