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MODELING OF STRESSES AND STRAINS IN CELL MEMBRANES SUBJECTED TO MICRO-INJECTION

PETER A. DJONDJOROV, KOSTADIN G. KOSTADINOV,
GEORGI I. STOILOV and VASSIL M. VASSILEV

Institute of Mechanics,Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 4, 1113 Sofia, Bulgaria

Abstract. This work is concerned with the determination of stresses and strains in cell membranes subjected to micro-injections. For that purpose, a suitable variational statement of the problem is developed within a continuum mechanics approach to the analysis of cell membrane geometry and physics. In this setting, the cell membrane is regarded as an axially symmetric surface in the three-dimensional Euclidean space providing a stationary value of the bending energy functional under the constraint of fixed total area. The Euler-Lagrange equations and the natural boundary conditions associated with the foregoing variational problem are derived, analyzed and used to express the stresses and moments in the membrane. Several examples of such surfaces representing possible shapes of cell membranes subjected to micro injection are determined numerically.

1. Introduction

Nowadays micro-injection is a common procedure in genetics, drug delivering, in-vitro fertilization, etc. During the process of a micro-injection, a micro pipette pierces the cell membrane and delivers substances within the cell interior. The success of a micro-injection to a large extent depends on the mechanical properties of the injected cell membrane and on the specific way of interaction between the injection pipette and the membrane.

Observing the literature on micro-injections of cells one realizes that large cells are the most often studied, typical examples being the zebrafish and mouse embryos. The analysis is mainly experimental, but several theoretical models have also been suggested (see, e.g. [1, 6, 11, 12, 16]).