Nineteenth International Conference on
Geometry, Integrability and Quantization
June 02–07, 2017, Varna, Bulgaria
Ivaïlo M. Mladenov and
Akira Yoshioka, Editors
Avangard Prima, Sofia 2018, pp 91–104
doi: 10.7546/giq-19-2018-91-104

Geometry, Integrability and Quantization XIX

PROJECTIVE BIVECTOR PARAMETRIZATION OF ISOMETRIES IN LOW DIMENSIONS

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Abstract. The paper provides a pedagogical study on vectorial parameterizations first proposed by O. Rodrigues for the rotation group in $\mathbb{R}^3$ by means of the so-called Rodrigue's vector. Although his technique yields significant advantages in both theoretical and applied context, the vectorial interpretation is easily seen to be completely wrong and in order to benefit most from this otherwise fruitful approach, we put it in the proper perspective, namely, that of Clifford's geometric algebras, spin groups and projective geometry. This allows for a natural generalization and straightforward implementations in various physical models, some of which are pointed out below in the text.

MSC: 17B81, 22E70, 81R05
Keywords: Geometric algebras, projective geometry, rotations, special relativity, vectorial parametrization

1. Rodrigues' Vector from 1840's Perspective

It may be considered a historical misfortune that the French - Jewish banker Olinde Rodrigues proposed his vector-parameter description of rotations as early as in 1840 (see [23]), three years before William Hamilton figured out quaterions, six years before Arthur Cayley came up with his famous transform and before W. K. Clifford and Sophus Lie were even born. And as it appeared so early, it did not find the appropriate context, namely that of hypercomplex numbers Lie groups and geometric algebras, and thus was not well enough understood and appreciated. From the 1840's perspective, the Rodrigues' construction could be realized via Euler's trigonometric substitution from the so-called spherical vector $\mathbf{s} = \varphi\mathbf{n}$, where φ denotes the rotation angle and $\mathbf{n}$ - the unit vector along the invariant axis (with