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ALTERNATIVE DESCRIPTION OF RIGID BODY KINEMATICS AND QUANTUM MECHANICAL ANGULAR MOMENTA

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Abstract. In the present paper we investigate an alternative two-axes decomposition method for rotations that has been proposed in our earlier research. It is shown to provide a convenient parametrization for many important physical systems. As an example, the kinematics of a rotating rigid body is considered and a specific class of solutions to the Euler dynamical equations are obtained in the case of symmetric inertial ellipsoid. They turn out to be related to the Rabi oscillator in spin systems well known in quantum computation. The corresponding quantum mechanical angular momentum and Laplace operator are derived as well with the aid of infinitesimal variations. Curiously, the coefficients in this new representation happen to depend only on one of the angles, which simplifies the corresponding system of ODE's emerging from separation of variables. Some applications of the hyperbolic and complex analogues of this construction in quantum mechanics and relativity are considered in a different paper cited below.
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Introduction

Finding new representations of an old problem may seem like a triviality, but quite often proves to be extremely fruitful both from a practical and theoretical point