DEFORMATION OF THE POISSON STRUCTURE RELATED TO THE ALGEBROID BRACKET OF DIFFERENTIAL FORMS AND APPLICATION TO REAL LOW DIMENSIONAL LIE ALGEBRAS

ALINA DOBROGOWSKA, GRZEGORZ JAKIMOWICZ, MARZENA SZAJEWSKA and KAROLINA WOJCIECHOWICZ

Institute of Mathematics, University of Białystok, Ciolkowskiego 1M, 15-245
Białystok, Poland

Abstract. The main goal of this paper is to present the possibility of application of some well known tools of Poisson geometry to classification of real low dimensional Lie algebras.

MSC: 53D17, 37K10
Keywords: Bi-Hamiltonian structure, deformation of Poisson structure Lie algebra, Lie algebroid, Lie-Poisson bracket

1. Introduction

The main purpose of the paper is to present how to apply Poisson and algebroid formalisms and some of their modifications to the theory of classification of real low dimensional Lie algebras. We show that it is also useful for calculating the Casimir functions corresponding to these algebras. In our considerations, we limit ourselves only to certain constructions from the area of Poisson geometry. So we only consider the vertical lift of the Poisson vector fields from Poisson manifold \((M, \pi)\) to its tangent bundle \((TM, \pi_{TM})\). Then we use them to build new Poisson tensors on manifold \(TM\) and this way to obtain the classes of bi-Hamiltonian manifolds.

The paper is organized as follows. At the beginning we recall the well known results about the Poisson manifold, the bi-Hamiltonian structure, the infinitesimal deformations of Poisson tensor and the Lie algebroid. Next we present some deformations of Poisson structures using the bi-Hamiltonian structure or the concept of