FUNDAMENTAL DOMAINS OF DIRICHLET FUNCTIONS

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Abstract. The concept of fundamental domain, as defined by Ahlfors, plays an important role in the study of different classes of analytic functions. For more than a century the Dirichlet functions have been intensely studied by mathematicians working in the field of number theory as well as by those interested in their analytic properties. The fundamental domains pertain to the last field, yet we found a lot of theoretic aspects which can be dealt with by knowing in detail those domains. We gathered together in this survey paper some recent advances in this field. Proofs are provided for some of the theorems, so that the reader can navigate easily through it.

MSC: 30C35, 11M26
Keywords: Dirichlet series, Euler product, fundamental domain, Riemann hypothesis, Riemann type of functional equation

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