SOLUTIONS TO A VECTOR HEISENBERG FERROMAGNET EQUATION RELATED TO SYMMETRIC SPACES

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Abstract. In this report we consider a vector generalization of Heisenberg ferromagnet equation. That completely integrable system is related to a spectral problem in pole gauge for the Lie algebra $\mathfrak{sl}(n+1, \mathbb{C})$. We construct special solutions over constant background using dressing technique.

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1. Introduction

In [8], the authors of the current text introduced the following matrix system of completely integrable equations

$$i u_t + \left[ (uQ_mu^\dagger Q_n)_x - u(Q_mu^\dagger Q_n)u_x \right]_x = 0 \quad (1)$$

and the corresponding auxiliary spectral problem. Above, the subscripts mean partial differentiation, “$\dagger$” denotes Hermitian conjugation and $Q_m, Q_n$ are diagonal matrices of dimension $m$ and $n$ respectively having $\pm 1$ on their principal diagonals. It is also assumed that the $n \times m$ matrix $u(x,t)$ fulfill certain algebraic condition, see [8] for more details.

System (1) contains as particular cases the classical $1+1$ dimensional Heisenberg ferromagnet equation, known to be integrable through inverse spectral transform [2, 6] and some of its integrable generalizations recently studied [1, 9, 10]. Its Lax