An Optimal System of One-dimensional Symmetry Lie Algebras of Coupled Nonlinear Schrödinger Equations

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Abstract. This paper is devoted to finding an optimal system of one-dimensional subalgebras of an eight-dimensional Lie algebra of point symmetry transformations, admitted by a system of two coupled nonlinear Schrödinger equations.

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INTRODUCTION

We consider a system of two coupled nonlinear Schrödinger equations (CNSEs)

$$\begin{aligned} iA_x + \frac{1}{2}A_{tt} + (|A|^2 - |B|^2)A = 0 \\ iB_x - \frac{1}{2}B_{tt} + (|B|^2 - |A|^2)B = 0 \end{aligned} \tag{1}$$

where $A(x,t)$ and $B(x,t)$ are complex functions of the real variables x, t. The system (1) has infinitely many motion invariants [1], reflecting the possibility of linearizing it by means of the inverse scattering method. The property of being integrable characterizes the CNSEs (1) as one of the systems having soliton solutions. In order to reveal this, one has to employ the theory of generalized symmetries, proving the existence of an infinite number of these symmetries.

In the present paper, however, we do not deal with the generalized symmetries. We consider only the finite dimensional case of the so called point symmetry transformations. Our main objective is to build up an optimal system of one-dimensional subalgebras of the most general Lie algebra of such symmetries.

Let G be a Lie group with Lie algebra $\mathfrak{g}$ and the adjoint representation of G on $\mathfrak{g}$ be denoted by $\mathrm{Ad}g(X)$, $g \in G$, $X \in \mathfrak{g}$. The adjoint representation introduces a conjugation (equivalence) relation in the set of all subalgebras of $\mathfrak{g}$ with the same dimension. An optimal system of subalgebras of $\mathfrak{g}$ is a list of not conjugate subalgebras, such that any other subalgebra of the same dimension is conjugate to a unique member of the list under some action of the adjoint representation. Constructing an optimal set of one-dimensional subalgebras is needed for finding and classifying the group-invariant solutions of differential equations.

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