Preface

This volume contains selected papers based on the talks, presented at the Conference 'Integrability, Recursion Operators and Soliton Interactions', held in Sofia, Bulgaria (29–31 August 2012) at the Institute for Nuclear Research and Nuclear Energy of the Bulgarian Academy of Sciences. Included are also invited papers presenting new research developments in the thematic area. The Conference was dedicated to the 65-th birthday of our esteemed colleague and friend Vladimir Gerdjikov. The event brought together more than 30 scientists, from 6 European countries to celebrate Vladimir's scientific achievements. All participants enjoyed a variety of excellent talks in a friendly and stimulating atmosphere. The main topics of the conference were those where Vladimir has contributed enormously during his career: integrable nonlinear partial differential equations, underlying algebraic and geometric structures of the integrable systems, soliton solutions, soliton interactions, quantum integrable systems, discrete integrable systems and applications of the nonlinear models. The papers, included in this volume will be useful to researchers with interests in these areas.

The organizers would like to express their gratitude to all participants and authors for their contribution to the success of the conference, and to Prof. Dimitar Tonev, director of the INRNE and the deputy director Prof. Lachezar Georgiev for their support.

Last, but nor least important, the organizers are grateful to Prof. Ivaïlo Mladenov and Dr. Mariana Hadjilazova for their help in the preparation of this volume.

Boyka Aneva
Georgi Grahovski
Rossen Ivanov
Dimitar Mladenov

Dear participants in the IROSI conference,
Dear friends and colleagues,

It is a great pleasure to welcome you here in Sofia at our Institute. With this conference we are celebrating the 65-th birthday of our esteemed colleague Vladimir Gerdjikov and the achievements of his active, fruitful and interesting career.

Vladimir was born on 7-th January 1947 in Stara Zagora, Bulgaria. He has studied in the Physics Department of the University of Sofia during the period 1964-1969. He did his Master Thesis at Göteborg Techniska Högskolan, Sweden, under the supervision of Professor Karl-Erik Eriksson. He defended it at the end of 1969 at the Faculty of Physics of Sofia University. In the 1970–71 Vladimir has been appointed as a physicist at the Institute of Physics of the Bulgarian Academy of Sciences, Sofia. His career path after that is

1971–1974
   Graduate student at Leningrad State University, USSR, where in 1974 he defended his PhD thesis *"On the infrared singularities of the Quantum Electrodynamics"* supervised by L. D. Faddeev and P. P. Kulish from the Leningrad Division of Steklov Institute of Mathematics;

1975–1976
   Research associate at INRNE, Sofia;

1977–1983
   Research associate and senior research associate at the Laboratory of Theoretical Physics at the Joint Institute for Nuclear Research, Dubna, USSR;

1984–1996
   Research associate and senior research associate at INRNE;

1987
   Doctor of Science with a Thesis *"Generating Operators of Soliton–Type Nonlinear Evolution Equations, Related to the Semisimple Lie Algebras"* defended at the Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research, Dubna, USSR;

1996
   Full Professor at INRNE, Sofia;

2002
   Founder and Head of the Laboratory of solitons, coherence and geometry at the Institute for Nuclear Research and Nuclear Energy.

Professor Gerdjikov has been very successful in his administrative work as well. He worked very actively as a member of the General Assembly of the Bulgarian Academy of Sciences (2004–2008) as a member of the Scientific council of the

Bulgarian Academy of Sciences (2004-2008) and as a Deputy Director of the Institute (2005–2008).

Vladimir Gerdjikov is an author of more than 190 scientific publications, including: more than 100 papers in scientific journals; more than 90 published reports in proceedings of international conferences; one monograph and co-editor of 5 proceedings volumes; more than 1000 independent citations, organizer and co-organizer of 13 conferences, workshops and symposia.

His main fields of research and achievements will be presented and celebrated at this conference, which I'm sure will be very fruitful and interesting.

I like very much his talent and enthusiasm to work with the young people. Indeed, he has supervised to completion 6 PhD students. This is one of the best results achieved in the Institute by a research supervisor! He has supervised also several MSc and BSc diploma theses.

All these results have been obtained by a tremendous amount of work. For his achievements Gerdjikov as a team leader has been awarded the INRNE Prizes for best work in theoretical physics two times: in 1998 and 2007.

Gerdjikov is a person who is addicted to work in a good sense, in a sense that he enjoys his work. Being always busy is something typical for his style, this is a feature which makes him unique. However this is possibly the only way of achieving such profound results - both in terms of quantity and quality.

I am very glad that many colleagues from abroad have gathered here for Vladimir's celebration. I am glad that here are also several young colleagues, some of whom as organizers of this nice event.

I would like to give Vladimir an official address as a memory of this special occasion.

I wish him many more successful years ahead in his career, more students, more conferences and symposia and to continue to be as energetic as ever!

Prof. D. Tonev, PhD
Director of INRNE
Sofia, August 29, 2012

Professor Vladimir S. Gerdjikov. Short Biography

Vladimir Gerdjikov was born on 7th January 1947 in Stara Zagora, Bulgaria. He obtained his higher education from the Physics Faculty of the University of Sofia during 1964-1969. Among his lecturers were several distinguished Bulgarian scientists such as Professors Christo Christov, Ivan Zlatev, Angel Nikolov, Alipi Mateev, Assen Datsev. Vladimir then prepared his Master Thesis in 1969 at Göteborg Techniska Hogskolan, Sweden, under the supervision of K.E. Eriksson, and defended it the same year at the Theoretical Physics Department of the Physics Faculty of the University of Sofia.

His first appointment was during the years 1970-71 as a physicist at the Institute of Physics of the Bulgarian Academy of Sciences, Sofia, Bulgaria.

During 1971-74, Vladimir Gerdjikov was a graduate student at Leningrad State University and in 1974 he completed and successfully defended his PhD thesis *"On the Infrared Singularities of the Quantum Electrodynamics"* at the Physics Department of Leningrad State University, Leningrad, USSR. His research was supervised by Ludvig D. Faddeev and Peter P. Kulish from the Leningrad Division of Steklov Institute of Mathematics.

Following his return to Sofia, during 1975-76 Vladimir Gerdjikov worked as a research associate at INRNE. Later on (1977-83), he moved to the Joint Institute for Nuclear Research, Dubna, USSR where he was a research associate and senior research associate at the Laboratory of Theoretical Physics. In 1987 he visited Dubna for three months and defended his Doctor of science thesis *"Generating Operators of Soliton Type Nonlinear Evolution Equations, Related to the Semisimple Lie Algebras"*.

These two long term visits (Leningrad 1971–1974 and Dubna 1977–1983) formed to a greater extent the scientific tastes and abilities of Vladimir. Being in Dubna he used the good chance to renew his collaboration with the Leningrad Division of Steklov Institute and especially with P. Kulish, but now on a new topic - integrable systems, both classical and quantum, and inverse scattering method. This topic was being developed also in Dubna by V. Makhan'kov, V. Gerdt and others. Besides, at the end of 1970's Vladimir Zakharov started a seminar on soliton theory in Moscow. This gave Vladimir an excellent opportunity not only to follow the

latest achievements in the field, but also to start personal contacts with many of the members of Zakharov's school, one of the leading groups in the world in the area of nonlinear sciences, including Sergey Manakov, Alexander Mikhailov, Evgeny Kuznetsov and others.

Also in Dubna he had the chance to meet Francesco Calogero and Marco Boiti. Although brief, these two meetings demonstrated common scientific interests and in fact led to a long term informal collaboration. Indeed, in 1985 Boiti invited him for three months to Lecce University, which started the Italian 'period' which still goes on. Another very important factor was the series of NEEDS biannual conferences which were held in Gallipoli, near Lecce, and where Vladimir was a regular participant.

The series of informal, but regular visits of Vladimir to Italy, besides Lecce University, included also Salerno University (G. Vilasi, G. Sparanno, M. Salerno), University of Naples (G. Marmo), University of Rome (F. Calogero, A. Degasperis, O. Ragnisco) and others.

Let me now briefly outline the topics to which Vladimir has contributed.

His PhD thesis deals with the infrared singularities in quantum electrodynamics. He proved that the quasiclassical results of Faddeev and Kulish hold true also for the quantum case. Thus, the infrared divergencies and the infrared singularities can be removed if instead of the naked charged particles, one considers their coherent states.

His next field became the soliton theory and the integrable models.

The idea that the inverse scattering method is a generalized Fourier transform started from the famous AKNS paper. Kaup was the first who realized that this fact is based on the completeness relation of the 'squared solutions' of the 2×2 Lax operator L and derived it. Soon after that, Gerdjikov and Khristov proposed an alternative rigorous proof of Kaup's result. Further development of this field initiated by Gerdjikov and collaborators included explicitly the gauge covariant formulation of the 'squared solutions' and the expansions over them. Thus, it became possible to derive all fundamental properties of the relevant soliton equations on the same footing. These expansions are actually spectral decompositions of the recursion operators $\Lambda_\pm$ which generate both the class of soliton equations and their Hamiltonian hierarchy. In a number of cases this approach allows one to derive the action-angle variables. Most of these results are summarized in the monograph with Gaetano Vilasi and Alexandar Yanovsky *"Integrable Hamiltonian Hierarchies"* published by Springer 'Lecture Notes in Physics' in 2008.

The expansions over the 'squared solutions' and their completeness relations were generalized in several directions. The first one concerned the Lax operators, which are quadratic 2×2 bundles. They allow one to solve the hierarchy containing

the derivative nonlinear Schroödinger equation and several of its generalizations, including the well known Gerdjikov-Ivanov (GI) equation.

The second one considered the generalized Zakharov-Shabat (GZS) systems of $n \times n$ Lax operators. Using such Lax operators one is able to solve the N-wave equations, the multicomponent nonlinear Schroödinger equation (MNLS) equations and their gauge equivalent. It is important here to introduce the 'squared solutions' using the fundamental analytic solutions (FAS) of L.

The third one concerned the Caudrey-Beals-Coifman (CBC) system whose FAS and 'squared solutions' are constructed in a substantially more elaborated way than for GZS. In the last years CBC systems with $\mathbb{Z}_h$ and $\mathbb{D}_h$-symmetries were treated and the completeness of their 'squared solutions' was also proved. Using it one can derive the action-angle variables for the 2-dimensional Toda field hierarchy.

In the early 1990's Vladimir started working, together with Ivan Uzunov, on another topic – N-soliton interactions in adiabatic approximation. In 1996 they derived a dynamical system for the $4N$ soliton parameters. Soon after that, a simplification of this system was proposed, which is known now as the complex Toda chain. An important consequence of this is the possibility to predict the asymptotic behavior of N-soliton trains. Since then this model has been tested a number of times and has been shown to describe adequately the soliton interactions not only for the nonlinear Schrödinger equation, but also of its vector generalization – the Manakov model. Both these equations find numerous applications in nonlinear optics, plasma physics, hydrodynamics etc.

His ability to organize research activities and collaborations culminated in 2002 when, together with several other colleagues, Vladimir founded the Laboratory of Solitons, Coherence and Geometry at the Institute for Nuclear Research and Nuclear Energy, and he has acted as the Head of the Laboratory ever since then. The scientific activities of the Laboratory, besides the soliton theory and soliton interactions, was extended to problems in quantum mechanics, differential geometry and the theory of coherent states. Since 2006 the Laboratory was joined by DSc Nikolay Kostov (1956-2011), who contributed very much to the study of multicomponent soliton equations and their reductions. Several classes of special reductions of the N-wave equations and the MNLS equations related to simple Lie algebras were analyzed. These include several special MNLS equations describing Bose-Einstein condensates.

During the last years Vladimir started another trend for constructing new types of integrable interactions based on the Riemann-Hilbert Problem with canonical normalization combined with Mikhailov's reduction group.

Vladimir was an organizer and co-organizer of several conferences, workshops and symposia. Most recently he was a co-organizer of the two international workshops on Complex structures, Integrability and Vector Fields together with Stancho Dimiev and Kouei Sekigawa (2008, 2010) held in Sofia. Together with Boyka Aneva and Georgi Grahovski he organized the International conference "Symmetries and Integrability of Difference Equations", SIDE-9 held in 2010 in Varna, Bulgaria.

He achieved remarkable results as a teacher and supervisor. He supervised 6 successful PhD dissertations, defended at various institutions: JINR, Dubna, USSR, INRNE and Cergy Pontoise University, Paris, France. He was a supervisor of several MSc and BSc theses as well. It is not an exaggeration to say that he has created a school of scientists, raised under his guidance, advice and mentorship.

Vladimir has designed and delivered two advanced lecture courses - on Soliton Theory and on the Theory of Lie Groups and Lie algebras for final year students and young researchers. He still works very actively in the area of integrability, shares his ideas and experience with younger colleagues, travels widely and collaborates with many scientists around the world.

In conclusion, I would like to wish our teacher, mentor, colleague and friend Vladimir Gerdjikov a long life, good health, a lot of energy and enthusiasm, and many more wonderful scientific ideas and results. I am sure that Vladimir will continue to be a source of inspiration, optimism and creativity to everyone who knows him.

Rossen Ivanov
Dublin Institute of Technology
Dublin Ireland

Opening speech
Dear Colleagues and Friends,

During the opening I heard a lot of nice words about myself, and I could not believe they were all true. It is impossible for one person to do so much work all by himself. And if I succeeded, that was because there were a lot of colleagues and friends helping me.

First of all I would like to thank Rossen Ivanov and Georgi Grahovski who are former graduate students of mine, for the idea to organize this conference. To say frankly, in the beginning I was reluctant, because it is not such a big deal to get that old. After all I agreed, not because of myself, but because such an event would be beneficial to my colleagues, friends and future students who may come around.

I am grateful to the Directorate of the Institute: Professor Dimitar Tonev and his deputy Professor Lachezar Georgiev for the support that made the conference possible.

Next I am grateful to my teachers at the University and mostly to academician Khristo Khristov. His lectures introduced not only me, but most of my colleagues in the field of theoretical and mathematical physics. I learned a lot from Peter Kulish and Ludvig Faddeev during the three years in Leningrad, and from Evgeny Khristov during the years I spent in Dubna.

The results in my papers wouldn't have been possible without co-authors, and I was lucky to work with a number of young people eager to understand and learn. So it is natural that I start with my former graduate students, noting briefly the results they are responsible for:

- Michail Ivanov – the GI equation;
- Alexander Yanovski – the gauge covariant approach to the recursion operators;
- Yordan Vaklev (1942 – 1999) and Michail Ivanov – the gauge covariant difference evolution equations;
- Evstati Evstatiev – his timely and precise observation lead us to the discovery of the complex Toda chain;
- Rossen Ivanov – reductions of N-waves, complete integrability of Camassa-Holm equation and CTC related to semisimple Lie algebras;
- Georgi Grahovski – reductions of N-waves, real Hamiltonian forms of 2-dim Toda's and many others;
- Tihomir Valchev, Victor Atanasov – soliton solutions to MNLS and their reductions, applications to Bose-Einstein condensates.

Thanks are due also to my co-authors who were able to put up with my character and still produce good results:

- Marco Boiti, Flora Pempinelli – Wadati-Konno-Ichikawa equation;
- Gaetano Vilasi and Alexander Yanovsky – for long years of collaboration during which we completed the monograph;
- Ivan Uzunov for proposing the study of N-soliton interactions and David Kaup, Evstati Evstatiev, Georgi Diankov for analyzing and testing the CTC;
- Nikolay Kostov (1956 – 2011) came up with most of the ideas for the papers on Bose-Einstein condensates (BEC);
- Giuseppe Marmo for patiently teaching me differential geometry, Hamiltonian dynamics that lead to the construction of real Hamiltonian forms jointly with Gaetano Vilasi and Assen Kyuldjiev;
- Mario Salerno, Vladimir Konotop, Bakhtiyor Baizakov, Victor Enol'skii – modulational instability of periodic solutions to MNLS and BEC;
- Evgeny Doktorov (1947 – 2011), Natalia Matsuka, Jianke Yang – CTC for derivative NLS;
- Rossen Dandoloff, Nikolay Kostov (1956 – 2011), Georgi Grahovski – differential geometry and Manakov model;
- Adrian Constantin, Rossen Ivanov – Camassa-Holm equation;
- Alexander Mikhailov, Tihomir Valchev, Georgi Grahovski – for deriving new types of soliton equations – the GMV equation;
- Radha Balakrishnan, Rossen Dandoloff, Dimiter Pushkarov, Avadh; Saxena – for organizing and editing the proceedings of the conference on "Topical Issue on Geometry, Integrability and Nonlinearity in Condensed Matter Physics", Bansko (2001);
- Milcho Tsvetkov and Plamen Fiziev – for organizing and editing the proceedings of the conference dedicated to Georgi Manev (1984 – 1965);
- Ivaïlo Mladenov, Stancho Dimiev, Kouei Sekigawa, Yasuo Matsushita – for organizing and editing the proceedings of two conferences on CSIVF in 2008 and 2010;
- Boyka Aneva, Plamen Iliev, Vassilis Papageorgiou. for organizing and editing the proceedings of SIDE-9 conference (2010) as special issue of SIGMA.

Special thanks to

- Francesco Calogero, Marco Boiti, Flora Pempinelli, Giusseppe Marmo, Gaetano Vilasi, Andrey Pogrebkov, Boris Konopelchenko, Barbara Prinari for giving me the chance to participate in a number of conferences on solitons;

- Ivaïlo Mladenov, Mariana Hadzhilazova, Akira Yoshioka for organizing the traditional conferences on Geometry, Integrability and Quantization in Varna.

Thus, I was able to enjoy south Italy and the Black sea and to have fruitful discussions with many famous scientists!

Finally and most of all I thank my parents and my family!!

In 1976 I married my wife Svetla, who at that time was working at Dubna. In 1977 I arrived to Dubna as a husband of my wife. That was crucial, because:

- I lost my freedom but gained a beautiful daughter and a son;
- I lost a bet to Evgeni Khristov but gained a teacher and a co-author;
- I had the chance to meet Vladimir Zakharov and his team in Moscow and Chernogolovka;
- I completed most of my DSc thesis and later defended it.

Best wishes to all of you.

Vladimir Gerdjikov
INRNE, Bulgarian Academy of Sciences
Sofia, Bulgaria