INÖNÜ-WIGNER CONTRACTION
OF KINEMATICAL GROUP REPRESENTATIONS

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Abstract

We study the contraction of the discrete series of Anti-de Sitter groups by exploiting the geometry of coadjoint orbits. Using the Kostant-Kirillov orbit method, we show that the representations we contract are explicitly realized on reproducing kernel Hilbert subspaces of the usual Bargman space. The contraction is performed by restricting elements of these subspaces on a common subvariety of a family of coadjoint orbits associated to the discrete series. The resulting representations are the massive representations of positive energy of the Poincaré group.

1. INTRODUCTION

Until recently, only contractions of Lie algebras and their representations were studied intensively.[1,2,3] Special cases have appared in the physics literature[3,4] and many of these examples are summarized in Ref. 5. However, global contractions and consequently the behavior of the unitary irreducible representations of their associated Lie groups under contractions have not been studied as thoroughly. The works of Michelson and Niederle[6] and of Dooley and Rice[7,8,9] are some notable exceptions in this direction. In fact, the first proper definition of the contraction of the unitary representations of Lie groups is given in 1972 by Michelson and Niederle.[6] The contraction of the principal series representations of the de Sitter groups $SO_0(n, 1)$ to representations of the Euclidian groups $E(n) = SO(n) \otimes_s \mathbb{R}^n$ and to the n-dimensional Poincaré group $\mathcal{P}^{n-1,1} = SO_0(n-1, 1) \otimes_s \mathbb{R}^n$ is then established. Here, $E(n)$ is a contraction of $SO_0(n, 1)$ along the subgroup $SO(n)$ and $\mathcal{P}^{n-1,1}$ is the contraction of $SO_0(n, 1)$ along its subgroup $SO_0(n, 1)$. However, this definition[6] is not general, but works very well for contracting principal series representations of noncompact semisimple Lie groups to representations of semidirect product groups obtained from the latter by the Inönü-Wigner contraction process. This is not surprising because of the existence of strong analogy between the two types of representations as described by Mackey.[10]

More recently, in 1983, Dooley and Rice[7,8,9] have shown that the irreducible representations of a Cartan motion group $K \otimes_s V$ associated with a Riemannian symmetric pair (G, K),[21] where K is a subgroup of a given Lie group G and V a vector space, are